ORDRE, CHAOS, FRACTALES

CHAOS AND FRACTALS

Lecturers: **Christophe BAILLY, Didier DRAGNA**

| Lecturers : 22.0 | TC : 0.0 | PW : 0.0 | Autonomy : 10.0 | Study : 0.0 | Project : 0.0 | Language : MI

Objectives

The concept of deterministic chaos has profoundly changed the way we approach the modeling of many problems. Poincaré's three body problem in celestial mechanics and Lorenz' work in meteorology are two now famous emblematic examples. The course introduces the main ideas and theoretical notions used to describe the behavior of these chaotic, nonlinear dynamical systems. A small number of effective degrees of freedom is very often sufficient to observe chaos, which makes the mathematical analysis affordable. The field of application was historically rather that of mechanics, but all fields of physics and even beyond (biology, medicine, economics, social sciences) are concerned, as will be illustrated in the course as well as in the case

Keywords : Dynamical systems, stability, bifurcations, limit cycle, strange attractor, chaos, fractal dimensions, Lyapunov exponents, control, identification and reconstruction.

Programme

1-Introduction to dynamical systems. 2-Stability of equilibria. Lyapunov stability, fixed points, limit cycles, Poincaré-Bendixon theorem, canonical bifurcations, attractor. 3-Fractals: introduction, generation, percolation, dimensions. 4-Sensitivity to initial conditions: introduction, Lyapunov exponents for maps, Lyapunov exponents for dynamical systems, long-time prediction. 5-Chaos in Hamiltonian systems: illustration in celestial mechanics, two- and restricted three-body problems; some properties of Hamiltonian systems, resonances, KAM theorem, stability of the solar system. 6-Control of chaos: motivation, algorithms and illustrations. 7-Identification and reconstruction from time series.

Learning outcomes

- To know the fundamental concepts for studying dynamic systems
- To appropriate these concepts through simple numerical case studies
- Be able to explain the behavior of nonlinear dynamic systems

Independent study

Objectifs : Master course concepts by solving case studies (to be done in Matlab or Python). Study cases: van der Pol oscillator, 2-D prey-predator model, fractals of Newton, chaos game, Julia and Mandelbrot fractals, logistic map and Sharkovsky theorem, ...

Méhodes : ... Henon strange attractor, Lorenz attractor (weather, synchronization and encryption), Rössler attractor, chaotic mixing, restricted three-body problem, Henon map, reconstruction of Lorenz and Rössler systems, application in medicine.

Core texts

Strogatz, S.H., *NONLINEAR DYNAMICS AND CHAOS (2ND EDITION)*, Westview Press, 2015
Alligood, K., Sauer, T., Yorke, J. *CHAOS: AN INTRODUCTION TO DYNAMICAL SYSTEMS*, Springer, 1996
Ott, E.*CHAOS IN DYNAMICAL SYSTEMS*, Cambridge University Press, 1993

Assessment

Knowledge 50%, Know-how 50%;
Knowledge = 40% Final written exam + 60% Assigned homework;
Know-how = 40% Final written exam + 60% Assigned homework;